

# Shell model calculations with modified empirical Hamiltonian in the $^{132}\text{Sn}$ region

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**Abstract.** Using recent experimental information for the  $^{132}\text{Sn}$  region, an empirical Hamiltonian is obtained by some modifications of a Hamiltonian (CW5082) originally derived from the  $^{208}\text{Pb}$  region. Shell model calculations with the new Hamiltonian show a remarkable improvement in the predictive power when compared with the available experimental results. It overcomes many limitations of the CW5082 Hamiltonian in this region, specially for  $N \geq 84$  isotones. The calculated level spectra and  $B(E2)$  values with the new Hamiltonian, also compare well with the available results calculated with the CD-Bonn and SKX Hamiltonians, reflecting consistency in the wave function structure at least in the low-lying regions. An interesting behaviour of effective charges is revealed in this region. It is shown that a drastic reduction of proton effective charge is necessary for obtaining  $B(E2)$  values for the  $N = 84$  isotones. Neutron effective charge is found to be in the range  $(0.62-0.72)e$ . We predict the spectroscopic properties of  $^{135,136}\text{Sn}$  not yet known experimentally. Further improvement of the modified Hamiltonian is also initiated.

**PACS.** 21.60.Cs Shell model – 21.10.-k Properties of nuclei, nuclear energy levels – 23.20.-g Electromagnetic transitions – 27.60.+j  $90 \leq A \leq 149$

## 1 Introduction

Few-valence-particle neutron-rich nuclei above the strongest doubly closed  $^{132}\text{Sn}$  [1] are interesting for many reasons. They provide the opportunity to extract empirical  $N$ - $N$  interaction, as well as to test the theoretical shell model description of nuclear structure in this region. Structure properties of some of these nuclei are important inputs for astrophysical r-process model calculations.

In a previous attempt [2], we have studied some  $N = 82-85$  isotones in this region with two  $(1+2)$ -body nuclear Hamiltonians, namely, KH5082 and CW5082, using the shell model code OXBASH [3]. The KH5082 and CW5082 are described in detail in the work of Chou and Warburton [4]. One important observation of our study [2] was that both the interactions, especially the CW5082 worked reasonably well in predicting binding energy, level spectra and electromagnetic properties of nuclear states for some  $N = 82-83$  isotones in the  $^{132}\text{Sn}$  region. But for  $N \geq 84$  isotones, the agreement of the calculated values with those from experiments was poor. We have pointed out that, particularly, the neutron  $(\nu)$ - $\nu$  two-body matrix elements (tbmes) of these interactions might be in-

appropriate and this has also been supported by some other very recent experimental and theoretical investigations [5, 6]. Thus the KH5082 and thereby CW5082 Hamiltonians, obtained by  $A^{-1/3}$  scaling from the Kuo-Herling Hamiltonian for the stable  $^{208}\text{Pb}$  region after some realistic modifications, though work for the  $^{132}\text{Sn}$  region within limitations, must be changed on account of the relatively large neutron richness in the  $^{132}\text{Sn}$  region. Sn, Sb isotopes above the  $^{132}\text{Sn}$  core are already about 10–11 neutrons away from their corresponding last stable isotopes.

We have initiated [7] to modify the existing nuclear Hamiltonian CW5082 in the light of the recent data for this region. We have obtained a remarkable improvement with a modified interaction by changing a few neutron-neutron and neutron-proton matrix elements and report in this article some of the interesting results. Further improvement in the modified Hamiltonian is indicated by changing also a few proton-proton matrix elements.

## 2 Formalism: model space and modified Hamiltonian

It has been mentioned above that the CW5082 predicts binding energies, level spectra and other properties for

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$50 \leq Z \leq 55$  and  $N < 84$  nuclei reasonably well. So in the present attempt, we have modified the CW5082 interaction. We assume  $^{132}\text{Sn}$  as the inert core. The valence space consists of five proton orbitals,  $\pi [1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2}]$  and six neutron orbitals  $\nu [1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2}]$ , with energies in MeV,  $\pi [0.0 (-9.6629) [8], 0.9624, 2.4396, 2.6972, 2.7915]$  and  $\nu [1.5609, 0.0 (-2.4553) [8], 2.0046, 0.8537, 1.6557, 2.6950]$ , respectively. Here we have replaced all the single-particle energies (spes) of the proton and neutron orbitals in CW5082 by experimentally determined ones [9] (except  $\pi 3s_{1/2}$  and  $\nu 1i_{13/2}$  spes).  $\pi 3s_{1/2}$  spe is obtained from the local systematics [4] and  $\nu 1i_{13/2}$  is taken from Urban *et al.* [10].

Five proton-neutron tbmes of the CW5082 interaction were obtained by Chou and Warburton by adjusting them to reproduce the energies of  $I^\pi = 0^-$  and  $1^-$  levels of  $^{134}\text{Sb}$ . It is important to note that a recent precise measurement of the binding energy of this  $0^-$  state by Fogelberg *et al.* [2,8] has changed the previous value [11] by a significant amount of about 200 keV. The recently measured [8] binding energy of  $^{132}\text{Sn}$  is also different from the earlier value [11]. So the  $\nu$ - $\pi$  tbmes also need modification to incorporate these important changes.

We change the neutron-neutron and proton-neutron tbmes keeping the proton-proton tbmes the same as those in CW5082. The six  $\nu$ - $\nu$  diagonal tbmes with  $I^\pi = 0^+$  were already noted to be too attractive in the work of Chou and Warburton [4]. We multiply all these six tbmes by a factor of 0.48. This factor is obtained by reproducing the binding energy of  $^{134}\text{Sn}$  ( $-6.365$ ). All the binding energies (in MeV) are with respect to  $^{132}\text{Sn}$  [8]. Three excited states in  $^{134}\text{Sn}$  [12], predominantly from the  $(\nu 2f_{7/2})^2$  multiplet, at energies 725.6, 1073.4, and 1247.4 keV are used to modify the  $\langle \nu 2f_{7/2}^2 | V | \nu 2f_{7/2}^2 \rangle^{2^+, 4^+, 6^+}$  tbmes. The  $\langle \nu 1h_{9/2} 2f_{7/2} | V | \nu 1h_{9/2} 2f_{7/2} \rangle^{8^+}$  tbme has been changed to reproduce the energy of the  $8^+$  level at 2508.9 keV.

Similarly, using the binding energy ( $-12.952$ ) and the  $1^-, 2^-, 3^-, 4^-, 7^-, 8^-, 10^+, 9^+, 10^-, 11^-, 12^-$  excited levels at energies 13.0, 330.7, 383.5, 554.8, 283.0 [2], 1073, 2434, 2126, 4094, 4425 and 4517 keV respectively, of  $^{134}\text{Sb}$  [13], we have modified 12 dominant proton-neutron tbmes. Thus we have changed only ten  $\nu$ - $\nu$  tbmes and twelve  $\nu$ - $\pi$  tbmes to obtain the SMN5082 Hamiltonian from CW5082.

From the results of shell model calculations with the CW5082 interaction it was found that its proton-proton tbmes were reasonable. So in obtaining SMN5082 the  $\pi$ - $\pi$  tbmes have not been changed. Recently, the two-valence-proton nucleus  $^{134}\text{Te}$  [14] has been studied more extensively, compared to  $^{134}\text{Sn}$  and  $^{134}\text{Sb}$  nuclei. It is expected that the inclusion of new experimental data of  $^{134}\text{Te}$  will further improve the interaction (SMN5082). We have initiated this effort by using a few  $^{134}\text{Te}$  data. We have used the binding energy of  $^{134}\text{Te}$  ( $-20.56$ ) and its three excited levels  $2^+, 4^+$  and  $6^+$  at energies 1279, 1576 and 1692 keV [14], respectively, predominantly from the  $(\pi 1g_{7/2})^2$  multiplet, to modify only four important  $\pi$ - $\pi$

**Table 1.** Comparison of calculated and experimental [8] binding energies in MeV with respect to  $^{132}\text{Sn}$ .

| Isotope                | Expt. <sup>(a)</sup><br>[8] | Theoretical           |                       |        |
|------------------------|-----------------------------|-----------------------|-----------------------|--------|
|                        |                             | SMN                   | SMPN                  | CW5082 |
| $^{134}\text{Te}_{82}$ | 20.560(26)                  | 20.643                | 20.560 <sup>(b)</sup> | 20.512 |
| $^{135}\text{I}_{82}$  | 29.083(25)                  | 29.303                | 29.055                | 29.102 |
| $^{136}\text{Xe}_{82}$ | 39.003(25)                  | 39.368                | 38.977                | 39.103 |
| $^{137}\text{Cs}_{82}$ | 46.419(24)                  | 46.911                | 46.247                | 46.582 |
| $^{134}\text{Sb}_{83}$ | 12.952(52)                  | 12.952 <sup>(b)</sup> | 12.952                | 12.768 |
| $^{135}\text{Te}_{83}$ | 23.902(93)                  | 23.990                | 23.913                | 23.624 |
| $^{136}\text{I}_{83}$  | 32.861(55)                  | 32.983                | 32.753                | 32.505 |
| $^{137}\text{Xe}_{83}$ | 43.029(25)                  | 43.482                | 43.124                | 42.863 |
| $^{134}\text{Sn}_{84}$ | 6.365(104)                  | 6.363 <sup>(b)</sup>  | 6.363                 | 6.705  |
| $^{135}\text{Sb}_{84}$ | 16.565(113)                 | 16.989                | 16.989                | 17.017 |
| $^{136}\text{Te}_{84}$ | 28.564(55)                  | 28.975                | 28.907                | 28.860 |
| $^{137}\text{I}_{84}$  | 37.934(37)                  | 38.336                | 38.131                | 38.011 |
| $^{135}\text{Sn}_{85}$ | 8.437(401) <sup>(c)</sup>   | 9.053                 | 9.053                 | 8.926  |
| $^{136}\text{Sb}_{85}$ | 19.516(301) <sup>(c)</sup>  | 20.306                | 20.306                | 19.759 |
| $^{137}\text{Te}_{85}$ | 31.775(122)                 | 32.410                | 32.345                | 31.762 |
| $^{136}\text{Sn}_{86}$ | 12.208(501) <sup>(c)</sup>  | 13.041                | 13.041                | 13.162 |
| $^{137}\text{Sb}_{86}$ | 23.257(401) <sup>(c)</sup>  | 24.477                | 24.477                | 24.144 |
| $^{137}\text{Sn}_{87}$ | 14.280(600) <sup>(c)</sup>  | 16.046                | 16.046                | 15.179 |

<sup>(a)</sup> Errors are within parentheses.

<sup>(b)</sup> Fitted.

<sup>(c)</sup> From systematics [11].

tbmes of SMN5082 and the resulting interaction is named as SMPN5082.

We shall refer to, for brevity, SMN5082 and SMPN5082 as SMN and SMPN, respectively. Thus, of the 2101 tbmes of the CW5082 Hamiltonian, we have changed only 22 tbmes for SMN and additional four proton-proton tbmes for SMPN.

## 2.1 Results and discussions

With these new interactions, we have calculated binding energies, level spectra and  $B(E2)$  values for some  $N = 84-87$  Sn, Sb, Te and I isotopes. Two different calculations have been carried out with these two slightly differing interactions. It is seen that even with this minor change, the result for the binding energies with the SMPN interaction is consistently better than that of SMN. So we compare the binding energies calculated with both the interactions. It should be noted that the results for Sn and Sb isotopes should be the same for both the interactions. For  $^{136}\text{Te}$  and  $^{137}\text{Te}$ ,  $^{137}\text{I}$  the results are expected to be slightly different with SMN and SMPN. The level spectra with the SMN interaction for  $^{135}\text{Sb}$  and  $^{137}\text{I}$  have been compared in (figs. 1(a), (c)) with the experimental as well as that of the parent CW5082, to show the improvement. In figs. 1(b) and (d) we give for comparison the experimental level spectra and those calculated with SMN and SMPN interactions for  $^{136-137}\text{Te}$ .

It is emphasised that the new SMN and SMPN interactions produce consistently good results also for the

|   |   |  |   |   |  |
|---|---|--|---|---|--|
| (a) $^{135}\text{Sb}$   |   |  | (b) $^{136}\text{Te}$   |   |  |
| $23/2^+$ 1898<br>$17/2^+$ 1564<br>$19/2^+$ 1305<br>$15/2^+$ 1124<br>$11/2^+$ 711<br>$5/2^+$ 690<br>$7/2^+$ 0<br><b>SMN</b>  | $23/2^+$ 1971<br>$17/2^+$ 1475<br>$19/2^+$ 1343<br>$15/2^+$ 1118<br>$11/2^+$ 707<br>$5/2^+$ 282<br>$7/2^+$ 0<br><b>Expt.</b>  | $17/2^+$ 2074<br>$19/2^+$ 1840<br>$15/2^+$ 1590<br>$11/2^+$ 1066<br>$5/2^+$ 618<br>$7/2^+$ 0<br><b>CW5082</b>  | $17^-$ 5283<br>$15^-$ 4888<br>$17^-$ 5216<br>$(15^-)$ 4794<br>$15^-$ 4820<br>$9^+$ 3718<br>$3^-$ 3405<br>$12^+$ 3284<br>$9^-$ 3198<br>$10^+$ 3168<br>$10^+$ 2818<br>$8^+$ 2189<br>$6^+$ 1399<br>$4^+$ 1084<br>$2^+$ 654<br>$0^+$ 0<br><b>SMN</b>                | $(17^-)$ 5160<br>$(15^-)$ 4794<br>$14^-$ 3720<br>$(10^-, 9^-)$ 3340<br>$12^+$ 3187<br>$10^+$ 2792<br>$8^+$ 2132<br>$6^+$ 1383<br>$4^+$ 1037<br>$2^+$ 607<br>$0^+$ 0<br><b>Expt.</b>   | $17^-$ 5216<br>$15^-$ 4820<br>$9^+$ 3705<br>$3^-$ 3399<br>$10^+$ 3343<br>$10^+$ 3281<br>$9^+$ 3173<br>$12^+$ 3179<br>$10^+$ 2821<br>$8^+$ 2172<br>$6^+$ 1396<br>$4^+$ 1085<br>$2^+$ 648<br>$0^+$ 0<br><b>SMPN</b>  |
| (c) $^{137}\text{I}$  |   |  | (d) $^{137}\text{Te}$   |   |  |
| $33/2^+$ 3790<br>$31/2^+$ 3658<br>$29/2^+$ 3173<br>$27/2^+$ 3071<br>$25/2^+$ 2812<br>$23/2^+$ 2262<br>$21/2^+$ 2053<br>$19/2^+$ 1507<br>$17/2^+$ 1382<br>$15/2^+$ 1116<br>$13/2^+$ 1051<br>$9/2^+$ 686<br>$11/2^+$ 606<br>$5/2^+$ 406<br>$5/2^+$ 255<br>$7/2^+$ 0<br><b>SMN</b> | $(33/2^+)$ 3853<br>$(31/2^+)$ 3652<br>$27/2^+$ 3085<br>$29/2^+$ 3167<br>$25/2^+$ 2743<br>$23/2^+$ 2223<br>$21/2^+$ 2038<br>$19/2^+$ 1609<br>$17/2^+$ 1313<br>$15/2^+$ 1109<br>$13/2^+$ 954<br>$11/2^+$ 620<br>$9/2^+$ 554<br>$5/2^+$ 243<br>$7/2^+$ 0.0<br><b>Expt.</b> | $33/2^+$ 4588<br>$31/2^+$ 4510<br>$27/2^+$ 3574<br>$29/2^+$ 3565<br>$25/2^+$ 3125<br>$23/2^+$ 2870<br>$21/2^+$ 2427<br>$19/2^+$ 1988<br>$17/2^+$ 1562<br>$15/2^+$ 1380<br>$13/2^+$ 1223<br>$9/2^+$ 853<br>$11/2^+$ 804<br>$5/2^+$ 455<br>$5/2^+$ 309<br>$7/2^+$ 0<br><b>CW5082</b> | $31/2^+$ 3979<br>$33/2^-$ 3606<br>$29/2^+$ 3416<br>$27/2^-$ 3063<br>$25/2^-$ 2953<br>$23/2^-$ 2506<br>$21/2^-$ 2177<br>$19/2^-$ 1783<br>$17/2^-$ 1589<br>$13/2^-$ 1238<br>$15/2^-$ 1148<br>$9/2^-$ 802<br>$11/2^-$ 642<br>$5/2^-$ 97<br>$7/2^-$ 0<br><b>SMN</b> | $(31/2^-)$ 3627<br>$(29/2^+)$ 3273<br>$(27/2^-)$ 3075<br>$(25/2^-)$ 2732<br>$(23/2^-)$ 2490<br>$(21/2^-)$ 1996<br>$19/2^-$ 1723<br>$17/2^-$ 1477<br>$15/2^-$ 1141<br>$(13/2^-)$ 1101<br>$11/2^-$ 608<br>$7/2^-$ 0<br><b>Expt.</b> | $31/2^-$ 3961<br>$33/2^-$ 3592<br>$29/2^+$ 3421<br>$27/2^-$ 3036<br>$25/2^-$ 2943<br>$23/2^-$ 2493<br>$21/2^-$ 2170<br>$19/2^-$ 1760<br>$17/2^-$ 1567<br>$13/2^-$ 1233<br>$15/2^-$ 1150<br>$9/2^-$ 787<br>$11/2^-$ 637<br>$5/2^-$ 93<br>$7/2^-$ 0<br><b>SMPN</b> |

**Fig. 1.** Comparison of calculated and experimental excitation energies for  $N = 84, 85$  isotones, (a)  $^{135}\text{Sb}$ , (b)  $^{136}\text{Te}$ , (c)  $^{137}\text{I}$ , (d)  $^{137}\text{Te}$ .

$N = 82-83$  isotones of these and Xe, Cs nuclei, which we have studied earlier with CW5082 and KH5082 [2].

The calculated binding energies (table 1) can be largely affected if the spes contain errors. The  $\pi 3s_{1/2}$  and  $\nu i_{13/2}$  spes might have uncertainties because of the ways they are determined. These have small effect for the ground states of the nuclei considered. The binding energies (table 1) are well reproduced with the two new Hamiltonians for  $N = 82, 83$  isotones. For  $N \geq 85$ , Sn and Sb nuclei, the binding energies quoted as experimental are derived from the local systematics and therefore have large errors (table 1). Therefore, it is difficult to draw a definite conclusion regarding the agreement of the calculated binding energies with experimental values for these nuclei. But for  $N = 84$  isotones and  $^{137}\text{Te}_{85}$ , slight over-binding can be noted for the calculated values. So we assume this over-binding to persist also for  $N \geq 84$ , Sn and Sb isotopes.

With the CW5082, the calculated binding energies are relatively less over-bound compared to the SMN and SMPN for  $N \geq 84$ . This is because the neutron spes of CW5082 were made less bound by adding 100 keV to each neutron spe. Our observation regarding this over-binding for  $N \geq 84$  is that it has an approximate systematic property. Over-binding, with SMPN, is  $\Delta \times (N - 82)$ ,  $\Delta = 0.1$  and  $0.175$  MeV for I and Te respectively. For  $^{135-137}\text{Sb}$  this is  $0.4(N - 83)$  MeV and that in  $^{135-137}\text{Sn}$ , is  $0.6(N - 84)$  MeV except for even-even  $^{136}\text{Sn}$ . The over-binding decreases with increasing valence proton number. This systematics as well as the detailed consideration of the interacting  $\pi-\pi$ ,  $\pi-\nu$  and  $\nu-\nu$  pairs in the valence space of the  $N \geq 84$  isotones suggest that the over-binding has a connection with the increasing neutron richness or  $N/Z$  ratio. The over-binding increases particularly with the increasing  $\nu-\nu$  pairs. This indicates the necessity of modification of more

$\nu$ - $\nu$  and  $\nu$ - $\pi$  tbmes beyond the dominant ones, which requires further data.

In figs. 1(a)-(d), we compare excitation spectra from our shell model calculations with CW5082, SMN and SMPN Hamiltonians with very recent experimental spectra. The agreement for all the nuclei is excellent, except for the 282 keV  $5/2^+$  level in  $^{135}\text{Sb}$ , showing the improvement achieved through the new Hamiltonians.

We compare our results for  $^{135}\text{Sn}$ , for which no experimental spectra are available yet, (except estimates from systematics for lowest few levels by Urban *et al.* [15]), with that predicted in ref. [6]. The energies with respect to the  $7/2^-$  ground state of the  $5/2^-$ ,  $3/2^-$ ,  $11/2^-$ ,  $9/2^-$ ,  $15/2^-$ ,  $3/2_2^-$ ,  $9/2_2^-$  and  $7/2_2^-$  levels are at 233, 353, 657, 701, 993, 1020, 1434 and 1535 keV in our calculation, whereas these levels are at 226, 356, 611, 706, 911, 643, 1093 and 1192 keV, respectively, in ref. [6]. The energies of the first five excited states of our calculation compare closely to the corresponding results of ref. [6] as well as with the estimates by Urban *et al.* [15]. But the result starts deviating by more than 200 keV for higher excited levels with the calculation of ref. [6].

Encouraged by the good agreement of our results with the available experimental data for all the nuclei, we predict also the spectra of  $^{136}\text{Sn}$ , an important nucleus for the r-process nucleosynthesis. The excitation energies with respect to the  $0^+$  ground state of the  $2^+$ ,  $4^+$ ,  $4_2^+$ ,  $6^+$ ,  $2_2^+$ ,  $5^+$ ,  $8^+$  and  $2_3^+$  levels are 578, 886, 994, 1086, 1106, 1272, 1682 and 1696 keV, respectively. It is interesting to compare our prediction for the energy of the first-excited  $2^+$  state at 578 keV with the estimate for it at about 600 keV from systematics by Urban *et al.* [15].

The excitation spectrum of  $^{135}\text{Sb}$  (fig. 1(a)) studied through prompt gamma ray emission up to  $23/2^+$  [16] is reproduced excellently in our shell model calculation using the new SMN Hamiltonian. But the  $5/2^+$  level at 282 keV [17] populated via beta decay of  $^{135}\text{Sn}$  is not reproduced in our calculation ( $E_{5/2^+} = 690$  keV). This behaviour is also noticed in other shell model calculations and have been discussed elaborately by Shergur *et al.* [17] and references therein.

In  $^{136}\text{Te}$  a long-lived isomeric  $12^+$  level was expected [18] from the analogy with  $^{212}\text{Po}$ , as well as from theoretical calculations [19]. But it was not seen experimentally. Its non-existence was confirmed [19] experimentally in an indirect way from the study of the neighbouring  $^{137}\text{I}$  nucleus. Our results in fig. 1(b) excellently reproduces the experimental data confirming that the observed  $12^+$  level at 3187 keV is indeed yrast and non-isomer. Thus the missing  $12^+$  isomer issue [19] is also resolved theoretically. The  $2_2^+$  level in this nucleus is estimated at 1568 keV from systematics [18], whereas our result is 1591 keV (SMPN) and 1603 keV (SMN). The position of the lowest  $3^-$  level in  $^{136}\text{Te}$ , which is of considerable interest [18] is also shown in fig. 1(b) to be at 3284 keV. This level, obviously involves many more unchanged tbmes of the original CW5082 and thus may have some uncertainty in the calculated energy.

The spectra of  $^{137}\text{I}$  [19] and  $^{137}\text{Te}$  [20] (figs. 1(c), (d)) show a kind of regularity indicating collectivity in their

excitations. Clear indication of signature splitting is seen in the spectra of both  $^{137}\text{I}$ ,  $^{137}\text{Te}$  nuclei. Our calculations reproduce this behaviour perfectly for  $^{137}\text{I}$ . But for  $^{137}\text{Te}$ , all the levels with signature  $-1/2$  (except for  $31/2^-$ ) are reproduced within  $\approx 50$  keV, whereas the levels with  $+1/2$  signature deviate by  $\approx 150$ – $200$  keV.

The structures of the wave functions of the  $^{137}\text{I}$  energy levels show large configuration mixing leading to such collectivity. For example, with SMN, for the  $5/2^+$  first-excited state, 13 configurations contribute 75% to the wave function and the rest is contributed by at least 25 other configurations. The most dominant  $(\pi 1g_{7/2})^3(\nu 2f_{7/2})^2$  configuration contributes 35.5% to the wave function. One can compare this with the structure of the first-excited state  $5/2^+$  in  $^{135}\text{I}_{82}$  for which the neutron shell is closed. Only 5 configurations contribute 98%, out of which 80% is contributed by the  $(\pi 1g_{7/2})^2(\pi 2d_{5/2})$  configuration. This state is, therefore, predominantly of single-particle nature.

In  $^{137}\text{Te}$  there is an estimate of the energy of the first  $9/2^-$  [15] level from the experimental systematics at around 700 keV above the  $7/2^-$  ground state. This is close to our results in fig. 1(d) (802 keV). Similarly, the estimated  $(5/2^-, 3/2^-)$  level at around 100 keV of [15] is most likely a  $5/2^-$  level as shown by our calculation in the same fig. 1(d) at 97 keV. Figures 1(c) and (d) help resolving ambiguous  $I^\pi$  of the levels in  $^{137}\text{I}$  and  $^{137}\text{Te}$ .

From the very good agreement of the calculated spectra with the experimental ones for all the nuclei compared here, one can hope that the new Hamiltonians, SMN and SMPN, can be useful for describing structure properties of at least low-lying yrast and near yrast states for all these few-valence-particle very neutron-rich nuclei in the  $^{132}\text{Sn}$  region.

To test the wave functions corresponding to the SMN Hamiltonian and to derive effective charges for this region, we have calculated the  $B(E2)$  values for the transitions  $2^+ \rightarrow 0^+$ ,  $4^+ \rightarrow 2^+$ ,  $6^+ \rightarrow 4^+$  and  $8^+ \rightarrow 6^+$  in  $^{134}\text{Sn}$  and obtained the values 73.2, 73.8, 36.5 and 4.8 in  $e^2\text{fm}^4$ , respectively, with neutron effective charge  $e_n^{\text{eff}} = 0.72e$ . (the  $B(E2)$  values are expressed throughout in  $e^2\text{fm}^4$ ). This value of effective charge  $e_n^{\text{eff}} = 0.72_{-0.08}^{+0.06}e$ , has been fixed by reproducing the experimental  $B(E2)$  value ( $36 \pm 7$ ) of the  $6^+ \rightarrow 4^+$  transition in  $^{134}\text{Sn}$  [21]. The corresponding  $B(E2)$  values in ref. [6] with  $e_n^{\text{eff}} = 0.70e$  are 70.1, 69.6, 35.8 and 4.9, respectively, showing very good agreement. Henceforth effective charges will be expressed in units of “ $e$ ”.

In table 2, we have compared the calculated and the available experimental  $B(E2)$  values [5,21] for  $^{134}\text{Sn}$ ,  $^{134,136}\text{Te}$  and  $^{135}\text{Sb}$  nuclei with different sets of effective charges. For  $^{135}\text{Sb}$ , only an approximate value for the half-life is found in the literature [9,16]. A  $B(E2)$  value  $\simeq 45$  has been extracted from the value of the half-life of the  $19/2^+$  level using the expression given in ref. [22] and the theoretical value of the internal conversion coefficient [23].

It is found that to reproduce the  $B(E2)$  value for the  $6^+ \rightarrow 4^+$  transition in  $^{134}\text{Te}$  a proton effective charge as low as  $1.34 \pm 0.01$  is needed. Whereas, for the  $0^+ \rightarrow 2^+$

**Table 2.** Comparison of calculated (with SMN) and experimental (error not available for Sb)  $B(E2)$  values for  $N = 84$  isotones using four sets of (proton, neutron) effective charges in the present calculation. Ranges of values for neutron and proton effective charges are discussed in the text. Note that the calculated  $B(E2)$  values for the  $0^+ \rightarrow 2^+$  transition in  $^{134}\text{Sn}$  are included for which no measurement has yet been reported.

| Isotope                     | $I_i^\pi \rightarrow I_f^\pi$ | $B(E2)$ in ( $e^2 \text{ fm}^4$ ) |                         |                    |                    |                    |             |                         |                    |
|-----------------------------|-------------------------------|-----------------------------------|-------------------------|--------------------|--------------------|--------------------|-------------|-------------------------|--------------------|
|                             |                               | Expt.<br>[5, 9, 16, 21, 24]       | SMN                     |                    |                    |                    | Theoretical |                         |                    |
|                             |                               |                                   | $e_p^{\text{eff}}$      | $e_n^{\text{eff}}$ | $e_p^{\text{eff}}$ | $e_n^{\text{eff}}$ | CW5082 [2]  | CD-Bonn [6]<br>SKX [17] |                    |
|                             |                               | $e_p^{\text{eff}}$ 1.47           | $e_n^{\text{eff}}$ 0.72 | 0.80               | 1.47               | 1.0                | 0.62        | 1.47                    | 1.55 [6], 1.5 [17] |
|                             |                               |                                   |                         | 0.72               | 0.0                | 0.62               | 1.00        |                         | 0.70               |
| $^{134}_{52}\text{Te}_{82}$ | $6^+ \rightarrow 4^+$         | $83.5 \pm 1.2$                    | 100.2                   | 29.7               | 100.2              | –                  | 97.1        | –                       |                    |
|                             | $0^+ \rightarrow 2^+$         | $960 \pm 120$                     | 869.8                   | 257.6              | 869.8              | –                  | 858.4       | 880 [6]<br>810 [17]     |                    |
| $^{136}_{52}\text{Te}_{84}$ | $0^+ \rightarrow 2^+$         | $1030 \pm 150$                    | 2165                    | 1043               | 975                | 1185               | 2989        | 2500 [6]<br>2300 [17]   |                    |
| $^{134}_{50}\text{Sn}_{84}$ | $6^+ \rightarrow 4^+$         | $36 \pm 7$                        | 37                      | 37                 | 0                  | 27                 | 97          | –                       |                    |
|                             | $0^+ \rightarrow 2^+$         | –                                 | 366                     | 366                | 0.0                | 271                | –           | –                       |                    |
| $^{135}_{51}\text{Sb}_{84}$ | $19/2^+ \rightarrow 15/2^+$   | $\simeq 45$                       | 84                      | 67                 | 5                  | 56                 | 89          | –                       |                    |

transition the corresponding proton effective charge is about  $1.54 \pm 0.10$ . It is important to note that the proton effective charge of the  $6^+ \rightarrow 4^+$  transition gives a  $B(E2)$  value for the  $0^+ \rightarrow 2^+$  less than the experimental lower limit for it. The  $B(E2)$  value for the  $6^+ \rightarrow 4^+$  transition appears to be more precise than that for the  $0^+ \rightarrow 2^+$  transition. With  $e_p^{\text{eff}} = 1.47$ , (a value in between, and used in the literature [25]),  $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$  as in ref. [2], the calculated  $B(E2)$  values for the  $6^+ \rightarrow 4^+$  and  $0^+ \rightarrow 2^+$  transitions in  $^{134}\text{Te}$  (table 2) compare well with the experimental values.

However, the  $B(E2)$  values with  $e_n^{\text{eff}} = 0.72$  and  $e_p^{\text{eff}} = 1.47$ , for the  $19/2^+ \rightarrow 15/2^+$  and  $0^+ \rightarrow 2^+$  transitions in  $^{135}\text{Sb}$  and  $^{136}\text{Te}$ , are 84 and 2165, respectively. These are about double the corresponding experimental values 45 and 1030. This kind of result is also obtained with other Hamiltonians (CD-Bonn and SKX) and have been discussed by Radford *et al.* [5], and Shergur *et al.* [17]. Recently, problems close to this have also been studied by Terasaki *et al.* [26] in a schematic model. They attributed the anomalous behaviour of the  $0^+ \rightarrow 2^+$   $E2$  transition in the  $^{136}\text{Te}$  to the reduction in neutron pairing above the  $N = 82$  magic gap. Thus, effective charges extracted from the pure proton ( $^{134}\text{Te}$ ) and neutron ( $^{134}\text{Sn}$ ) systems cannot reproduce the  $B(E2)$  values for the  $N = 84$ , Sb and Te nuclei.

Now, let us consider  $^{134}\text{Sn}$ ,  $^{134}\text{Te}$  and  $^{136}\text{Te}$ , keeping aside the  $N = 84$ ,  $^{135}\text{Sb}$  nucleus. For  $^{136}\text{Te}$ , if we use the proton effective charge 1.47 from  $^{134}\text{Te}$ , the  $e_n^{\text{eff}}$  needed is  $\simeq 0$ . This value is anomalously low compared to  $e_n^{\text{eff}}$  (0.72) for  $^{134}\text{Sn}$ . Similarly, one can use  $e_n^{\text{eff}}$  from  $^{134}\text{Sn}$  and vary  $e_p^{\text{eff}}$  to fit the  $B(E2)$  value of  $^{136}\text{Te}$ . In this case the  $e_p^{\text{eff}} \simeq$

0.80, which is anomalously low compared to that derived for  $^{134}\text{Te}$ .

But, if we include also  $^{135}\text{Sb}$  and consider the  $B(E2)$  values of three  $N = 84$  isotones of Sn, Sb and Te, the following observations can be made. We find that  $e_p^{\text{eff}} = 1.0$  and  $e_n^{\text{eff}} = 0.51 \pm 0.11$  reproduce the  $B(E2)$  value,  $1030 \pm 150$  for the  $0^+ \rightarrow 2^+$  transition in  $^{136}\text{Te}$ . The same set of effective charges gives  $B(E2) = 42_{-13}^{+14}$  for the  $19/2^+ \rightarrow 15/2^+$  transition in  $^{135}\text{Sb}$ , and this is close to the experimental value. Similarly, with  $e_p^{\text{eff}} = 1.0$  and  $e_n^{\text{eff}} = 0.54$ , which reproduce very closely the measured  $B(E2)$  value 45, for the  $19/2^+ \rightarrow 15/2^+$  transition in  $^{135}\text{Sb}$ , one gets a value 1070 for the  $0^+ \rightarrow 2^+$  transition in  $^{136}\text{Te}$  and is within the experimental range. But  $e_n^{\text{eff}}$  from these sets of effective charges gives about half of the experimental  $B(E2)$  value for the  $6^+ \rightarrow 4^+$  transition in  $^{134}\text{Sn}$ , (since  $e_n^{\text{eff}} = 0.72$ ). So for these  $N = 84$  isotones, only the proton effective charge  $\simeq 1.0$ , with the neutron effective charge in the range (0.62–0.72) can bring all the calculated  $B(E2)$  values close to the experimental limits. It may be found from table 2 (column 7) that the most reasonable choice for a single set of effective charges for  $N = 84$  isotones is  $e_p^{\text{eff}} = 1.0$  and  $e_n^{\text{eff}} = 0.62$ . With the SMPN Hamiltonian similar results are obtained.

So this drastic reduction of proton effective charge for  $N = 84$  isotones compared to the  $N = 82$  isotones is quite interesting and needs further detailed experimental and theoretical investigations.

In  $^{136}\text{Sn}$ , the  $\Delta v$  (seniority) = 0,  $6^+ \rightarrow 4^+$  transition between the pure  $(\nu 2f_{7/2})^4$  multiplet levels is expected to be severely inhibited, as in  $^{136}\text{Xe}$  [2, 22, 27]. With SMN, the  $(\nu 2f_{7/2})^4$  configuration is although dominant in the

$6^+$  and  $4^+$  levels in  $^{136}\text{Sn}$  (about 80% and 75.6%, respectively); yet due to configuration mixing the  $B(E2)$  value for this transition is not so severely inhibited. It is predicted to be 12.9 with  $e_n^{\text{eff}} = 0.62$ . The measurement for this  $B(E2)$  value will be very helpful to conclude whether a further reduction of neutron effective charge is needed in this  $N = 86$  Sn isotope.

### 3 Conclusion

In summary, our calculations with the SMN Hamiltonian obtained by modifying CW5082 show several interesting results for this very neutron-rich region. We have found good agreement with experimental data for the excitation spectra of all the nuclei except for the first  $5/2^+$  level at 282 keV in  $^{135}\text{Sb}$ . Our results clearly help in assigning spin and parity of some levels in these nuclei. Predictions for levels in  $^{135}\text{Sn}$  and  $^{136}\text{Sn}$  may motivate and provide a guidance for future experiments. Results for  $^{136}\text{Te}$  resolve theoretically the missing  $12^+$  isomer issue and predict the position of the  $3_1^-$  level in it. The calculations for the  $A = 137$ , I and Te, reveal signature of collectivity. The calculated  $B(E2)$  values with the usual  $e_p^{\text{eff}}$  around 1.47 for the  $^{134}\text{Te}$  isotope compare well with the very recent experimental data. But a drastic reduction in proton effective charge is required to reproduce the  $B(E2)$  values for  $N = 84$  isotones. The neutron effective charge seems to lie in the range (0.62–0.72). A further modification of SMN is initiated in SMPN by modifying a few  $\pi$ - $\pi$  tbmes. We are looking forward to change more  $\pi$ - $\pi$  tbmes in SMPN using some already existing data and hope to modify more tbmes and spes using further experimental information which will be available in the near future.

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